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ALGEBRAIC KNOTS ARE ALGEBRAICALLY DEPENDENT

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ABSTRACT. Algebraic knots are linearly dependent in G_-, the algebraic knot concordance group. An example of a linear relation between four algebraic knots is constructed.

An algebraic knot is any one component link of an isolated singularity of a complex curve. Such knots have been classified [**4**]. Rudolph [**3**] asked for a description of the subgroup of the knot concordance group generated by this class of knots, and whether or not these knots form an independent set. Litherland [**2**] showed that signature functions are not sufficient to rule out linear relations among them. In this note we will produce an example showing that no algebraic concordance invariants will suffice. We will prove:

PROPOSITION. *The algebraic knots are linearly dependent in G_-, the algebraic knot concordance group.*

1. Notation. A *satellite* knot is any knot in S^3 whose complement contains an essential torus. Fix an oriented satellite knot S and an essential torus T in the complement of S. Let V denote the solid torus bounded by T. Note that V contains S. The core of V, called the *companion* of S (associated with T) will be denoted by C. Define the *winding number* w of S of the homology relation $S \sim wC$ in V. Orient C so that $w \geqslant 0$ (there is a choice to be made when $w = 0$). Finally, set $E = f(S)$ where $f\colon V \to S^3$ is an orientation and longitude preserving embedding onto an unknotted solid torus in S^3. We shall call E the *embellishment* of S.

Set $\Lambda = Z[t, t^{-1}]$ and $\Lambda_0 = Q(t)$, the quotient field of Λ. For any oriented knot K, write A_K for the Alexander module of K, and B_K for the Blanchfield pairing on A_k (= linking pairing $A_k \times A_K \to \Lambda_0/\Lambda$). A_K has a square presentation matrix with entries in Λ. Any such matrix $A_K(t)$ is called the *Alexander matrix* of K. The associated matrix $B_K(t)$ for B_K (with entries in Λ_0/Λ) is called the associated *Blanchfield matrix*.

2. Example dependence relation. The following result is implicit in Kearton [**1**].

THEOREM. *Let S be a satellite knot with core C, embellishment E and winding number w. Then*

$$B_S(t) = B_E(t) \oplus B_C(t^w).$$

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EXAMPLE. Let $(p_1, q_1;\ p_2, q_2)$ denote the (p_1, q_1) cable about the (p_2, q_2) torus knot. (The winding number of this satellite knot is q_1 with respect to (p_2, q_2).) Let $J = (13,2;\ 3,2) \# (15,2)$ and $K = (15,2;\ 3,2) \# (13,2)$. According to **[4]** these are both connected sums of algebraic knots. (The (p, q) torus knot is always algebraic, and $(p_1, q_1;\ p_2, q_2)$ is algebraic if $p_1 > q_1 p_2 q_2$.) It follows from the above theorem that

$$B_J(t) = B_{(13,2)}(t) + B_{(3,2)}(t^2) + B_{(15,2)}(t) = B_K(t).$$

According to **[5]**, as J and K have the same Blanchfield pairing they are S-equivalent, and therefore algebraically concordant. Therefore, the four prime factors of J and K (which are distinct) satisfy a relation in G_-.

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